NWFP, PUBLIUCC SERVICE COMMISSION, PESHAWAR

COMPETITIVE EXAMINATION FOR PROVINCIAL MANAGEMENT SERVICE, 2008.

APPLIED MATHEMATICS, PAPER –II

Time Allowed 03 Hours **Max: Marks: 100**

NOTE: Attempt any **FIVE** questions selecting at least **TWO** questions from each section, each part carries **10** marks.

SECTION- A

Q.No.1 (a) Find a particular solution by method of variation of parameters of the differential equation $\frac{d^2y}{dx^2} + y = \tan x \ for \ -\frac{\pi}{2} < x < \frac{\pi}{2}$. Why the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$ is stipulated?

(b) Solve the following simultaneous differential equations

$$\frac{dx}{dt} + \frac{dy}{dt} + y - x = e^{2t}$$

$$\frac{d^2x}{dt^2} + \frac{dy}{dt} = 3e^{2t}$$

Q.No.2 (a) Solve the differential equation

$$\frac{d^2y}{dx^2} + y = 0 \text{ such that } y(0) = 0,\ y'(0) = 1$$

by the Power series method.

(b) Solve the boundary value problem

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,\ \ 0 \le x \le a;\ 0 \le y \le b$$

$$u(x,0) = f_1(x);\ u(x,b) = f_2(x)$$

$$u(0,y) = g_1(y);\ u(a,y) = g_2(y)$$

with the requirement that $u(x,y) \to 0 \ \ as\ y \to \infty$.

Q.No.3 (a) Find the second order partial differential equation with general solution is expressed in terms of arbitrary functions $\phi(x)\, and\, \psi(x)$,

$$z = \phi(y + ax) + \psi(y - ax).$$

(b) Solve the following homogenous problem

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2},\ \ \ 0 \le x \le \pi,\ \ t > \ 0,$$

subject to condition

$$u(x,0) = 0; \quad u_t(x,0) = 8\sin^2 x$$

$$u(0,t) = 0; \quad u(\pi,t) = 0$$

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SECTION- B

Q.No.4 (a) Compute the value of the integral

$$\int_0^1 \frac{1}{1+x^2}\,dx \ ; \quad n=6$$

using one-third Simpsons rule. Hence calculate the approximate value of π to 5 D.

(b) Use Newton-Raphson method to solve the equation $x\log_{10} x = 1.2$ work to 5D.

Q.No.5 (a) Solve the following system of equations by Gauss-Seidel method up to 3 D.

$$x_1 + 6x_2 - 3x_3 = 4$$
$$5x_1 + 2x_2 - x_3 = 6$$
$$2x_1 + x_2 + 4x_3 = 7$$

Comment why Gauss- Seidel method is faster than Jacobi method?

(b) The following data gives the percentage of criminals for different age groups

Age (Less than x)	25	30	40	50
% for criminals y	52	67.3	84.1	94.4

Using Lagrange formula, find the percentage of criminals under the age of 35.

Q.No.6 (a) Show that δ_j^i is invariant i.e. it has same components in every coordinate system.

(b) Let A_i^{ij} and B_q^{p} be two tensors then prove that $A_i^{ij} B_q^{p} = B_q^{p} A_i^{ij}$

Q.No.7 (a) Establish the relationship between the Kronecker delta and Levi-Civita tensors.

(b) A small firm manufactures necklaces and bracelets.The combined number of necklaces and bracelets that it can handle per days are 24. Each bracelet takes 1 hour of labor to make and each necklace takes a half hour. The total number of hours of labor available is 16. If the profit on the bracelet is \$2 and the profit on the necklace is \$1, how many of each product should be produced daily to have maximum profit.